11.1

$\lim_{n\to\infty} a_n$

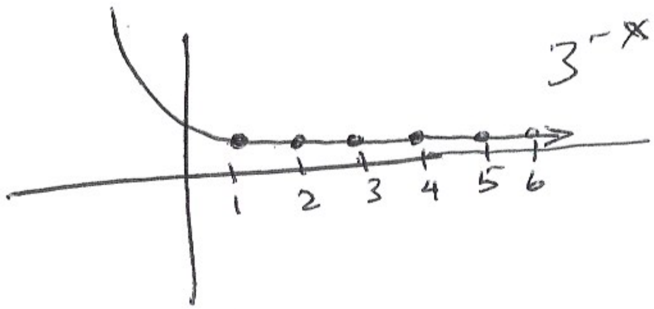
$\lim_{n\to\infty} 3^{-n} = 0$

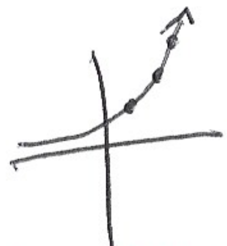
$\{3^{-n}\} = \{3^{-1}, 3^{-2}, 3^{-3}, \ldots\}$
$= \{\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots\}$

$f(n) = 3^{-n}, \quad n \in \mathbb{Z}^+$
$f(x) = 3^{-x}, \quad x \in \mathbb{R}$

$\lim_{x\to\infty} 3^{-x} = 0$

$\lim_{n\to\infty} 3^{-n} = 0$





$\lim_{x\to\infty} 3^{x} = \infty$

$\lim_{n\to\infty} 3^{n} = \infty$

NOTE: THE LIMIT DOES NOT EXIST

EXPLAINS WHY LIMIT DNE

IF $\lim\limits_{\underset{\uparrow}{\underset{\in \mathbb{R}}{x}} \to \infty} f(x) = \underset{\underset{\in \mathbb{R}}{\uparrow}}{L}$ THEN $\lim\limits_{\underset{\in \mathbb{Z}}{n} \to \infty} f(n) = L$

IF $\lim\limits_{x \to \infty} f(x) = \infty$ OR $-\infty$

THEN $\lim\limits_{n \to \infty} f(n) = \infty$ OR $-\infty$ (CORRESPONDINGLY)

NOTE: IF $\lim\limits_{x \to \infty} f(x) \neq \infty, \neq -\infty$

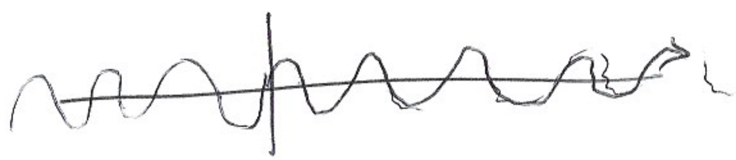
BUT DNE

THAT TELLS YOU NOTHING ABOUT $\lim\limits_{n \to \infty} f(n)$

CONSIDER $\{\sin n\pi\} = \{0, 0, 0, \dots\}$

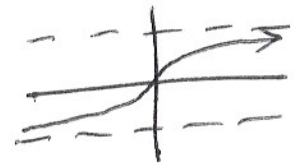
$\lim\limits_{n \to \infty} \sin n\pi = 0$

$\lim\limits_{x \to \infty} \sin x\pi = \lim\limits_{x \to \infty} \sin \pi x$ DNE



$a_n = \frac{1}{n} + 4\tan^{-1} n$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{n} + 4 \lim_{n\to\infty} \tan^{-1} n = 0 + 4\cdot\frac{\pi}{2} = 2\pi$$



$\leftarrow y = \frac{\pi}{2}$

$$\lim_{x\to\infty}\left(\frac{1}{x} + 4\tan^{-1} x\right) = \lim_{x\to\infty} \frac{1}{x} + 4 \lim_{x\to\infty} \tan^{-1} x$$

$$= 0 + 4\cdot\frac{\pi}{2}$$

$$= 2\pi$$

$$\lim_{n\to\infty} \frac{4n}{\sqrt{n^2+2}} = 4$$

$$\lim_{x\to\infty} \frac{4x}{\sqrt{x^2+2}} \cdot \frac{\frac{1}{\sqrt{x^2}}}{\frac{1}{\sqrt{x^2}}} = \lim_{x\to\infty} \frac{4}{\sqrt{1+\frac{2}{x^2}}}$$

$\frac{1}{\sqrt{x^2}} = \frac{1}{x}$

$\sqrt{x^2} = |x| = x$

SINCE $x \to \infty$

$$= \frac{4}{\sqrt{1+0}} = 4$$

$\uparrow$

$\frac{\infty}{\infty}$

INDETERMINATE

L'HÔPITAL'S RULE ?

$$\lim_{n\to\infty} \cos^{-1} e^{-n}$$

$$= \lim_{n\to\infty} \cos^{-1}(e^{-n}) = \frac{\pi}{2}$$

NOT MULTIPLICATION !

$$\lim_{x\to\infty} \cos^{-1}(e^{-x})$$

$$= \lim_{y\to 0} \cos^{-1} y \quad \leftarrow y \text{ REPLACES } e^{-x}$$

$$= \cos^{-1} 0 = \frac{\pi}{2}$$

$$\lim_{x\to\infty} e^{-x} = 0$$

$$\cos^{-1} x = \arccos x$$

$$\neq \frac{1}{\cos x}$$

$$(\cos x)^{-1}$$

$$\lim_{n\to\infty} e^{\sin n\pi} = e^{\lim_{n\to\infty} \sin n\pi} = e^{0} = 1$$

$$\lim_{n\to\infty} \sin n\pi = 0$$

SIMILAR TO MATH 1A

$$\lim_{x\to\infty} e^{\sin \pi x} \text{ DNE}$$

IF $\lim\limits_{n\to\infty} |a_n| = 0$ THEN $\lim\limits_{n\to\infty} a_n = 0$

CONSIDER $a_n = \dfrac{(-1)^n}{n^2}$

$$\lim_{n\to\infty} \frac{(-1)^n}{n^2}$$

$\lim\limits_{x\to\infty} \dfrac{(-1)^x}{x^2}$ DNE SINCE f DOESN'T EXIST FOR ~~MANY~~ INFINITELY MANY INFINITELY LARGE X

$x \in \mathbb{R}$

eg. $x = \frac{3}{2}$

$$(-1)^{\frac{3}{2}} = \sqrt[2]{(-1)^3} \text{ or } \left(\sqrt{-1}\right)^3 \notin \mathbb{R}$$

$$\lim_{n\to\infty} \left|\frac{(-1)^n}{n^2}\right| = \lim_{n\to\infty} \frac{1}{n^2} = 0$$

SO $\lim\limits_{n\to\infty} \dfrac{(-1)^n}{n^2} = 0$